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Garlick, Karen

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An investigation to evaluate the effectiveness of a primary school's improvement plan focusing on maths attainment and progress through problem solving.

Biography

The author is working as a teacher in Devon having achieved their PGCE. They have been working in the early years for over 20 years before gaining their teaching qualification. They hope to continue their study on to a Master's in Education

Introduction

Mathematics is a fundamental skill which is required throughout life; children need to develop a fluency to enable them to be confident in using maths (Myatt 2018), however, maths anxiety within the British adult population shows that there is a belief that maths is hard and only for those who are clever (Haydock and Manning, 2019). Internationally, the UK performs well in reading and science, yet does not do as well in mathematics (OCED, 2016). This lower attainment in maths within the UK is reflected within the case study school. This report will look at how problem solving can help to improve maths attainment in primary school pupils. I will begin by looking at Piaget's stages of development and his theories of when children can solve abstract problems and what other theorists have written. The literature review will then explore how teachers can facilitate problem solving in their class. The report will look at what the school's improvement plan says and how this has improved learning, linking to the literature discussed. Implications on the practice of the school and how I can develop my knowledge and information further will be discussed.

Aims and rationale

The case study school's improvement plan states that maths is a priority within the school due to the low data from the previous year. Their Ofsted report from 2016 reflects the need

to improve the teaching of maths throughout Key Stage 2. Mathematics has not always come easily to me; however, I do enjoy the logical nature of maths. I feel this gives me a perspective on mathematics and the struggles some children have in learning new concepts. This is why I have chosen to investigate mathematics in primary schools.

The numeracy plan created by the case study school states various strategies to improve the maths data throughout the school; in particular, it states the school will use role play areas in all classes to develop problem solving skills. Problem solving is defined by Skemp (1989) as a process where there is no set procedure to achieve a goal which requires adaptability. Observations and discussions at the case study school showed that the action plan is being implemented in the school and there is an improvement in the way the pupils and teachers view maths. There has also been an improvement in attainment and the predicted results for this year's SATs are higher. Despite this, the maths lead has noticed that children are still struggling to problem solve. This school improvement project will investigate how role play and problem solving can be used to improve mathematical understanding in a primary school.

According to Pound (2014:37) there are many complex theories developed by Piaget about how children develop which have been used to inform teaching practice today. One of his most notable theories is that of the stages of development. Piaget believed that children constructed and reconstructed their thoughts, building on their ideas by integrating their currently held ideas with new, more complex concepts. Children need to be at a particular stage of their development before they are able to learn new concepts (Pound, 2014:37). Piaget argued that abstract problem solving is achieved during the formal operations stage which begins around eleven years of age (Phillips, 1975). This is contradicted by the National Curriculum (2013) Key Stage 2 mathematics curriculum where it states, 'pupils should develop their ability to solve a wider range of problems, including increasingly

complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation.’ (DfE, 2013:30). Seeing that children only reach 11 years of age at the end of Key Stage 2; I will explore this conflict of ideas in the literature review.

Literature review

According to Pound (2014), Piaget believed that children within the concrete operational stage of development, from ages seven to eleven, are developing their logical thought, however, this is only through tangible resources. Once children enter the formal operational stage after twelve years of age, they can master logical and abstract thought (Pound, 2014). Donaldson (1978) expands on Piaget’s theories by explaining that in order to solve problems, children need to understand the question in a human sense. She called this embedded understanding. Disembodied or abstract thinking requires children to think beyond the human sense and how teachers can do this has not been fully explored. Donaldson (1978) asserts that Piaget’s theories are not accepted by a number of different educational theorists and ‘Piaget bashing’ is commonly seen in literature with claims that he underestimates what children can do (Doherty, 2009:12). Skemp (1989) argues that young infants have an innate ability to solve problems. However, he does state that the knowledge and understanding acquired by the natural enquiry of play is banked, stored for the future and is not shown straight away. Despite this, teachers are required to provide an opportunity for problem solving.

The National Curriculum (2013) requires children to solve problems throughout their time in primary education; problem solving is mentioned in the statutory and guidance parts of the curriculum. In an Ofsted report (2012:9), they identified that teachers and senior leadership were aware that pupils needed to improve problem solving and investigation

skills. Teachers were not giving enough time to problem solve for life, they were narrowly teaching these skills for tests. This is reiterated in Amanda Spielman's (2018) speech about the narrowing of the curriculum, in particular, English and Maths in upper Key Stage 2. Edwards-Leis and Robinson (2019) criticise the current curriculum for not specifying problem solving in its own element within the mathematics curriculum. Having looked at the requirements for problem solving in school, I will explore how problem solving can be included in the classroom.

In the influential Cockcroft report (1982), the importance of problem solving in mathematics was explored. Children should be given the opportunity to apply maths in an everyday context, especially revisiting concepts they have already learnt. If children are not given sufficient chance to develop these skills, they will not become proficient at solving problems, whether in maths, or in other areas of the curriculum. Contextual problems have now become common place in classrooms; however, the emphasis is on the skills required to solve the problem, rather than the problem solving (van Oers, 2014). Polya (1957) proposes a four-step model to solve problems to help teachers understand the process of problem solving. It includes understanding the problem, devising a plan, carrying out the plan, then evaluating how well the plan worked. Schico and Lawson (2004) suggest a fifth stage; finding a problem. They suggest children should have the chance to decide where there is a problem and whether a problem is worth solving. In order to be able to solve mathematical problems, children need to have a fluency in the fundamentals of mathematics (Myatt, 2018:184) and the knowledge of what mathematical information they need to know (Barton, 2018). The knowledge pupils need must be stored in the long-term memory by using low stake high challenge testing at regular intervals (Myatt 2018:185). However, Barton (2018) argues that fitting a real-life context into a maths problem can confuse students. He suggests that pupils can be guarded thinking they are being conned

into being engaged. He cites research which has influenced his belief; however, this research is over ten years old. Problem solving does not come easily to all and the difficulties faced by students will be explored next.

All children struggle with word problems (Atkins, 2016:85) and there are many different reasons why this happens. Atkins (2016) believes this is because children struggle with the language involved in explaining and solving the problem. Teachers often suggest children scan and highlight the word problem for specific words before answering the question. The rich English language means there is a huge list of words just for addition. Teachers need to be shifting their focus away from the product of investigations, to the process children use (Di Martino, 2018). Problem solving can be messy (Edwards-Leis and Robinson, 2019) but teachers should not fear losing control whilst facilitating problem solving with their pupils (Di Martino, 2018). These problems have been documented in primary school children; pre-schoolers contradict these findings as discussed next.

Rittle-Johnson and Siegler (1998) claim that pre-schoolers seem to have a more sophisticated understanding of numerical problem solving and this understanding declines in school-aged children. They suggest that this paradox is due to children in the early years having the opportunity to practice and apply the concepts needed, whereas, older children are not given the chance to practice using these concepts. Di Martino's (2018) research agrees, stating how pupils anticipate maths problems shows there is a difference between how younger children in kindergarten approach problems to older children in primary. He concludes that the more a child is exposed to mathematical problems, the more negatively they respond to problems. He suggests that this could be linked to students being worried about making mistakes. In Rittle-Johnson *et al.* (2016) more recent work, they state that school-aged children struggle to understand the symbolic forms in mathematics. One way early years differs from the rest of primary, especially Key Stage 2, is in the use of play to

provide learning experiences. I will look at the capacity play has in mathematics and problems solving.

Play provides a valuable engaging tool in problem solving (Edwards-Leis and Robinson, 2019); creativity comes from the flexibility of play (Pound and Lee, 2015). Co-operation is an essential part of play; problem solving in a group provides an opportunity to enhance others knowledge (Schiro and Lawson, 2004: 187). The National Curriculum purpose of study states, 'Mathematics is a creative and highly inter-connected discipline that has been developed over the centuries, providing the solution to some of history's most intriguing problems.' (DfE 2013:99). Role play gives children the chance to make sense of the world in which they live and there is a large amount of literature about the importance of play for children in the early years (DfE, 2017:9, Bruce, 2018); Worthington and Carruthers (2003) highlight the importance of using role play in children's mathematical development. However, there is little written about the importance of role play for older children (Grant and Mistry, 2010). Cast (2007, cited in Grant and Mistry, 2010) argues that role play can be hugely beneficial to children in Key Stage 2, allowing children to explore everyday experiences freely. However, it can be seen as a time filler if it is not used in a structured part of a lesson (Harrison *et al.*, 2005, cited in Grant and Mistry, 2010). Nevertheless, there is literature which explores the use of stories to help contextualise mathematical concepts.

Pound and Lee (2015:81) provides an example of year 5 pupils using a story to help visualise algebra, a complex abstract concept. Using this sense of story helps makes things more memorable. The ability to tell a story is innate in humans, dating back to when early humans told stories around a fire (Schiro and Lawson, 2004: vii); information can be organised and processed through oral storytelling (Daniel, 2012). This long history of using language to tell stories, rather than the relatively newer tool of written language, means that humans can learn better from oral stories (Barton, 2018:121). This inbuilt desire to tell

stories can be harnessed by the teacher as a tool to teach complex mathematical concepts. These stories and role play, when carefully planned, can provide a stimulus which can secure concepts (Myatt, 2018). The concrete, pictorial and abstract model of learning, which was inspired by Bruner's (1966) model of representation, can be enhanced through stories (Daniel, 2012:98). The structure of stories, comprising the components: causality, conflict, complications and character, can be used as a structure for teaching maths concepts (Barton, 2018:122). Placing problem solving into a real-life context reflects back to Donaldson's (1978) idea of presenting problems with a human sense, helping children to understand and conceptualise what is being asked. Piaget's theory that children cannot problem solve has been proved precarious in modern literature (Pound, 2014). Children need to have more opportunities to problem solve within a context, as identified by Donaldson (1978).

A critical evaluation of the case study school's work and progress to date and impact on children's learning

The attainment and progress of children's maths knowledge and skills are specifically named in the first priority of the school improvement plan. This shows how much emphasis the senior leadership team have put on the attainment of maths within the school. This target has been set because the data from the previous year had been low. The maths data from 2018 shows that the attainment in the early years was above the national average, with 79% of pupils achieving a good level of development. This means children gained the Early Learning Goals in the 3 prime areas of the Early Years Foundation Stage as well as in Literacy and numeracy. In Key Stage 1, the year 2 SAT's results showed that the children were above the national average in all areas, including maths, where 75% of the children achieved the expected level; one percentage mark below the national average. The Key Stage 2 results showed a significant difference to the national average. All areas

were low; however, maths was a lot lower than the national average with 33% of children achieving the expected standard and only 7% reaching greater depth. The school's latest Ofsted report from 2016 states that attainment in Key Stage 2 was low in mathematics despite good progress in Key Stage 1. Ofsted state in this report that 'the school recognises that it still has work to do on developing pupils' mastery in mathematics.' (Ofsted, 2016). Ofsted requires the school to plan and implement changes to their school improvement plan. Extracts taken from the case study school's website state that maths has been a school's priority for the last 4 years and that the current year 6's are on track to meet the national expectations this year. The school is a relatively small school, and this can cause an inaccurate skew in assessment data. The DfE have re-introduced 3-year average scale attainment scores for schools to negate the disproportionate effect individual results can have on data (DfE, 2018). High progress is shown by the year 6 girls, however, boys, pupil premium children and higher achievers show low progress. This data has been used to inform the school improvement plan and numeracy plan.

The school's improvement plan shows the priority the senior leaders have put on maths. It is the first priority within the plan, giving it the prominence it needs. This emphasis of putting maths to the forefront of the staff team's mind is reflected in the school's numeracy plan. Following conversations with the maths lead and class teacher, and observations during my experience in the year 5 and 6 class, I have witnessed the difficulties children show when solving word problems. Children are given the opportunity to solve word problems within a unit of work, complementing what they have already learnt, in line with the National Curriculum (2013). The school has recently introduced role play areas into every class, including the year 5 and 6 class I worked in. The role play areas are themed according to the class termly topic. Whilst I was there, the role play area was a World War 1 trench in the Autumn term and a Hogwarts' potions classroom in the Winter term. During

lessons, a specific group of children are given tasks separate to the rest of the class, which are related to the role play area. This can be done in any lesson, but it was seen to be popular amongst the children during maths. Maths problems would be set with a context relating to the topic or role play area. Organising World War 1 troops helped give a context to place value in hundreds, thousands and millions in line with the National Curriculum for year 5 (DfE, 2013) via the World War 1 topic the children were engaged in.

These observations reflect the numeracy action plan, which mentions role play and making maths fun for all children, a number of times. The discussions I have had with the maths lead did not specify whether the maths problems children had difficulty with were during lesson time or in the role play area. Due to limited time, children would more likely have to complete abstract word problems in lesson time rather than using the role play area. The belief the class teacher had about children struggling with word problems relates to Piaget's theory (Pound, 2014) regarding children not being developmentally ready to solve problems. The role play links to Donaldson's (1978) theory that children need a human sense to be able to solve problems, yet Donaldson does not include an age range to her development theories. There is a need for the senior leadership team to tackle how problem solving can be improved throughout the school. I will explore a few ways in which they could help children understand how to solve problems.

The suggestion from the literature, on solving mathematical problems by using stories to help teachers teach contextually complicated maths concepts, has not been observed in the case study school. It is possible stories have been used in the school, however, there were no discussions or observations linked to mathematical storytelling. This strategy has not been mentioned in the action plan but the immersive curriculum which is implemented by the headteacher would lend itself to using storytelling to teach complex mathematical concepts, even in Key Stage 2. Pound and Lee's (2015:81) suggestion that stories can

help children understand algebra concepts would provide another strategy the maths lead could use to develop problem solving skills in the school. The early years could provide another place for inspiration for the maths lead.

Literature indicates how strong problem solving is in the early years. Rittle-Johnson and Siegler (1998) comment on this paradox; younger children showing an ability to solve problems when older children struggle. Whilst the mathematics data at the school is not solely based on the ability to solve problems, there is a significant difference in the maths data between the early years and Key Stage 1 when compared to Key Stage 2. The literature shows how play can help conceptualise mathematical problems as well as oral storytelling. Could the learning through play culture we provide for the younger children provide better results for older children? It would be beneficial to the whole school if the maths lead could observe and research practice ideas in the early years and use this to inspire maths education in the school. Rittle-Johnson and Siegler (1998) suggest that younger children, being better at problem solving than older children, could be to do with their confidence in maths. It is evident in the action plan that the senior leadership have been working to improve maths confidence within and outside the classroom. I have observed this maths confidence within the school from both teachers and children.

The school improvement plan requires the senior leadership team to organise whole school training on the use of particular teaching methods ensuring consistency in teaching maths throughout the school. The plan states that maths will have a high focus throughout the school with whole school assemblies focusing on maths and displays in communal areas celebrating maths success in each class. I have observed an emphasis on making every second of the school day count and this was observed when teachers would give children literacy and numeracy challenges during parts of the day that are not traditionally teaching time, such as registration. These actions have been attributed to making children more

confident in maths inside and outside the classroom. This confidence has also been seen in the teacher's attitude towards their teaching. Despite this increase in confidence, the children are still finding problem solving difficult. As Myatt (2018) and Barton (2018) suggest, children need to have fundamental mathematical skills to enable them to solve problems.

The times table checks, mentioned in the numeracy plan, are carried out weekly in Key Stage 2. They provide a chance for the children to store mathematical knowledge needed to solve maths problems. These tests are a positive influence on the maths knowledge of all children, including those of a lower ability. Children celebrate each other's achievement and the effort that individuals have put into learning their times tables. The sense of competition is not between children but on individual progress. There is no mocking or belittling those who are on a lower ability test. Children are genuinely pleased for each other when they show they have progressed. There is a culture of celebrating progress for all children, regardless of their attainment. The times table checks are arranged into stages and once a child has completed a stage, they receive a certificate from the headteacher during an assembly. This positivity contributes to the maths confidence in children and has been shown to improve multiplication knowledge throughout Key Stage 2.

The improvement shown in maths attainment can be attributed to the increase in confidence the children show in maths, the ability to engage in meaningful role play involving maths and the chance to practice and test their fundamental maths skills in low stake testing. Yet despite this, teachers still observe a difficulty in solving problems, reflecting the literature. Implications for the school and my further professional development will be explored.

Potential implications for professional development and practice

The findings from the literature review show that young children struggle with dealing with abstract problems. Piaget theorises that children under the age of 11 cannot solve abstract problems (Pound, 2014) but when the children have a real-life context as part of the problem, they are more likely to be able to solve the problem (Donaldson, 1978). Teachers need to have an awareness of this developmental stage of children at primary age. As a teacher who is at the start of my career, I have the advantage of knowing this developmental stage of the children who will most likely be in my class. I would make sure the children are given contextual problems to solve, including chances to use role play to embed mathematical concepts. It could be said that the case study school do not have this particular knowledge of child development and had they understood this they may change their approach to problem solving in the classroom. Role play and immersive learning can be used in the facilitation of learning, yet teachers need to be careful not to try to create tenuous links, shoehorning a particular area of learning into any topic. Whilst the literature and observations in the case study school have shown that role play is effective at helping children to conceptualise this learning and solve problems, trying to make any topic or curriculum area into a problem to be solved could diminish the effectiveness of these learning models. The case study school use role play to use contextualise mathematical problems, but having an awareness of the developmental stage of children in their class could help them plan more effective problem solving.

The reading I have done regarding using storytelling to help teach maths and problem solving has shown that it is a very useful tool for teaching complex mathematical concepts. This approach to teaching maths has not been observed in practice, however, I have used storytelling to teach concepts to children in preschool. In the future, I will consider seeking out experiences to observe teachers who use storytelling to facilitate my professional development in this area. This approach to helping children to conceptualise mathematics

into real-life situations would line up with the school role play and immersive learning philosophy. This is something they could easily implement throughout the school which would help to improve maths outcomes.

The findings show that younger children are better at solving problems and may mean that the practice in early years helps children to understand the concepts they learn whilst in the early years. The early years philosophy of learning through play could be used and adapted to provide effective learning in older children. As an early years practitioner, with years of experience, I have the ability to adapt my practice of learning through play to suit the National Curriculum. However, the increase in understanding problems in the early years may be because the children are more confident in their abilities. I have observed multiple ways that schools can increase confidence and create a positive culture of celebrating success which is what I would hope to achieve in any future class I may have.

Conclusion

This report has explored mathematical problem solving within primary schools. A large proportion of the adult population in the UK are anxious about maths and this has trickled down through to our primary school pupils (Haydock and Manning, 2019), causing maths results to suffer in the UK education system (OCED, 2016). The maths data in Key Stage 2 at the case study school has been historically low compared to the rest of the country. Due to this data and their most recent Ofsted report, the school have placed maths as a high priority on their school's improvement plan. Whilst the plan has been improving attainment across the school, from discussions with the maths lead, children are still struggling to solve mathematical problems. This is why I had chosen to investigate mathematical problem solving in children.

This investigation found that whilst Piaget believed children under the age of 11 years are unable to solve abstract problems (Pound, 2014), Donaldson (1978) tells us that children can solve problems if they are given a human context to the problem. Role play has played a large part in the school, especially in maths which has shown to provide an effective learning approach. This could have contributed to the progress children have shown in their maths. The literature has also shown that storytelling can help children conceptualise complex mathematical concepts. The school could include this in their plan to help improve attainment even further.

The improvements the school has made to their maths curriculum has been shown to improve children's learning, but due to the size of this investigation, it is not possible to indicate how much impact these approaches have had. If this investigation were to be extended, I would examine the paradox Rittle-Johnson and Siegler (1998) discovered, that pre-schoolers were better problem solvers than older children and how this could improve practice for teachers working with children in Key Stage 2.

References

Atkins, S. L. (2016) *Creating a language-rich maths class*. London: Routledge.

Barton, C. (2018) *How I wish I'd taught maths*. Woodbridge: John Cat Educational Limited.

Bruce, T. (2018) 'The importance of play' in Trevarthen, C., Delafield-Butt, J. and Dunlop, A-W, eds. *The child's curriculum. Working with the natural values of young children*. Oxford: Oxford University Press. Pp. 39 – 58.

Bruner, J. S. (1966). *Toward a theory of instruction*. New York: W.W. Norton.

Cockcroft, W. H. (1982) *The Cockcroft Report: Mathematics count*. London: Her Majesty's Stationery Office.

Daniel, A. K. (2012) *Storytelling across the primary curriculum*. Oxon: Routledge.

Department for Education (2012) *Primary school accountability in 2018*. Crown copyright.

Available at:

https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/763614/Primary_school_accountability_technical_guide_2018_.pdf?fbclid=IwAR3mkGsDPH7X_TOAxmYN24nBOYLrHAw70YQzzzj0Lj-qDiTdvKied8iLDY [Accessed: 11 April 2019].

Department for Education (2013) *The national curriculum in England*. Crown copyright.

Available at:

https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/425601/PRIMARY_national_curriculum.pdf(Accessed: 10 April 2019).

Department for Education (2017) *Statutory framework for the early years foundation stage*.

Crown Copyright. Available at:

https://www.foundationyears.org.uk/files/2017/03/EYFS_STATUTORY_FRAMEWORK_2017.pdf (Accessed: 10 April 2019).

Di Martino, P. (2018) 'Pupils' views of problems: the evaluation from kindergarten to the end of primary school' *Educational studies in mathematics*. 100, pp. 291 – 307.

Doherty, M. J. (2009) *Theory of mind. How children understand others thoughts and feelings*. Hove: Psychology Press.

Donaldson, M. (1978) *Children's Minds*. Great Britain: Fontana Paperbacks.

Edwards-Leis, C. and Robinson, D. (2019) *Problem solving in primary mathematics*. Oxon: Routledge.

Grant, K. and Mistry, M, T. (2010) 'How does the use of role-play affect the learning of Year 4 children in a predominately EAL class?' *Education 3-13* 38 (2), pp. 155 - 164.

Haydock, D. and Manning, R. (2019) *Mathematics explained for primary teachers*. 6th edn. London: Sage.

Myatt, M. (2018) *The curriculum Gallimaufry to coherence*. Suffolk: John Catt Educational Limited.

OCED (2016) *Programme for international student assessment (PISA) Results from PISA 2015* Available at: <http://www.oecd.org/pisa/PISA-2015-United-Kingdom.pdf>

Ofsted (2012) *Mathematics: made to measure*. Available at: https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/417446/Mathematics_made_to_measure.pdf (Accessed: 10 April 2019).

Ofsted (2016) [Case study school] *Ofsted report*. Manchester: Ofsted.

Phillips, J. L. (1975) *The origins of intellect. Piaget's Theory*. 2nd edn. San Francisco: W.H. Freeman and Company.

Polya, G. (1957) *How to solve it: a new aspect of mathematical method*. 2nd edn. Princeton: Princeton University Press.

Pound, L. (2014) *How children learn*. London: Practical Pre-School Books.

Pound, L. and Lee, T. (2015) *Teaching mathematics creativity*. 2nd edn. Oxon: Routledge.

Rittle-Johnson, B. and Siegler, R. S. (1998) 'The relation between conceptual and procedural knowledge in learning mathematics: A review' in Donlan, C. ed. *The development of mathematics skills*. Hope: Psychology Press. Pp 75-128.

Rittle-Johnson, B., Fyfe, E. R. and Loehr, A. M. (2016) 'Improving conceptual and procedural knowledge: The impact of instructional content within a mathematics lessons.' *British Journal of Educational Psychology*, 86 (4), pp. 576-591.

Schiro, M. S. and Lawson, D. (2004) *Oral storytelling and teaching mathematics*. London: Sage.

Skemp, R. R. (1989) *Mathematics in the primary school*. London: Routledge.

Spielman, A. (2018) HMCI commentary: curriculum and the new education inspection framework. Available at: <https://www.gov.uk/government/speeches/hmci-commentary-curriculum-and-the-new-education-inspection-framework> (Accessed: 10 April 2019).

Van Oers, B. (2014) 'The roots of mathematising young children's play' in Kortenkamp, U., Brandt, B., Benz, C., Krummheuer, G., Ladel, S. and Vogel, R. (eds) *Early Mathematics Learning*. New York: Springer.

Worthington, M. and Carruthers, E. (2003) *Children's mathematics*. London: Paul Chapman Publishing.